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Global sea-level rise and its relation to the terrestrial reference frame

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Abstract We examined the sensitivity of estimates of global sea-level rise obtained from GPS-corrected long term tide gauge records to uncertainties in the International Terrestrial Reference Frame (ITRF) realization. A useful transfer function was established, linking potential errors in the reference frame datum (origin and scale) to resulting errors in the estimate of global sea level rise. Contrary to scale errors that are propagated by a factor of 100%, the impact of errors in the origin depends on the network geometry. The geometry of the network analyzed here resulted in an error propagation factor of 50% for the Z component of the origin, mainly due to the asymmetry in the distribution of the stations between hemispheres. This factor decreased from 50% to less than 10% as the geometry of the network improved using realistic potential stations that did not yet meet the selection criteria (e.g., record length, data availability). Conversely, we explored new constraints on the reference frame by considering forward calculations involving tide gauge records. A reference frame could be found in which the scatter of the regional sea-level rates was limited. The resulting reference frame drifted by 1.36 ± 0.22 mm/year from the ITRF2000 origin in the Z component and by -0.44 ± 0.22 mm/year from the ITRF2005 origin. A bound on the rate of global sea level rise of 1.2 to 1.6 mm/year was derived for the past century, depending on the origin of the adopted reference frame. The upper bound is slightly lower than previous estimates of 1.8 mm/year discussed in the IPCC fourth report.

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1 Introduction

Sea level change is considered a metric of paramount interest for climate change. It is an important observational constraint for global climate models. The observed sea-level change for the past 100 years has therefore been given considerable attention. Precise and near-global satellite altimetry data only provide information on the last 15 years or so (e.g., Beckley et al. 2007, 3.3 ± 0.4 mm/year for the period 1993–2007), a time span that is clearly too short to derive estimates for the rise in sea level on a century time scale. Tide gauges are the primary historical source of sea-level measurements over the past centuries, with rare records extending back to the early 18th century (Wöppelmann et al. 2008). The range of estimates for global sea-level rise derived from tide gauge records for the 20th century is rather wide, 1–2 mm/year (Bindoff et al. 2007). It is important to reduce this range to reveal a little more about the processes at work in sea level change. To a large extent the differences in estimates originate from the nature of the tide gauge measurement itself. Tide gauges measure sea level relative to a point attached to the land, hence the term relative sea level used to designate this quantity. The land can move vertically at rates comparable to the long-term sea-level signals expected from ocean thermal expansion and land-based ice melting. Thus, without independent estimates of vertical land motion, tide gauges cannot determine whether the sea level is rising or the land sinking (or both).

Advances in Geodetic technologies, and in particular the Global Positioning System (GPS) in continuous mode, have made it possible to obtain highly accurate measurements of vertical land motion at tide gauges, whether these are due to Glacial Isostatic Adjustment (GIA) or to other land motion processes (Teferle et al. 2006; Mazzotti et al. 2008; Wöppelmann et al. 2007, 2009). Wöppelmann et al. (2009) showed that using a global-scale, fully consistent processing strategy throughout the entire GPS data span considerably reduced technique errors and analysis artefacts, providing useful vertical velocities to account for land motion in tide gauge records. However, vertical velocity is a reference frame-dependent quantity, as were the average rates of global sea-level rise during the past century estimated by Wöppelmann et al. (2007, 2009) from GPS-corrected tide gauge records. These estimates represent changes in the height of the sea surface with respect to a conventional terrestrial (geocentric) reference frame, which is very sensitive to the origin and scale of the frame. It is now well recognized that an accurate reference frame is needed and that reference frame uncertainty affects mean sea level rise estimation (Nerem et al. 2000; Morel and Willis 2005; Beckley et al. 2007; Blewitt et al. 2010). Therefore, the resulting estimates depend on the adopted conventions of the reference frame and coordinate system, and the ability to obtain such a reference frame in practice (Kovalevsky et al. 1989).

The preferred reference frames for scientific applications that require a very high degree of accuracy are the realizations of the International Terrestrial Reference System (ITRS) which are named International Terrestrial Reference Frames (ITRF). The two latest, ITRF2000 and ITRF2005, are assumed to be expressed with respect to the Center of Mass (CM) of the entire Earth, including the oceans and atmosphere. Both origins were constrained using Satellite Laser Ranging (SLR) data, which is recognized as the most accurate technique for CM determination. However, these two frames differ by 1.8 ± 0.3 mm/year for the drift in origin in the Z component, and 0.5 ± 0.3 mm/year (0.08 ± 0.05 ppb/year) for the radial scale change (Altamimi et al. 2007). Such differences mostly explain the different rates of observed global sea-level rise obtained by Wöppelmann et al. (2007, 2009), 1.3 ± 0.3 and 1.6 ± 0.2 mm/year, respectively. Improving our understanding of sea-level rise and variability, as well as reducing the uncertainties associated with the estimates of change, critically depend on our ability to realize a stable terrestrial reference frame. The accuracy of the origin and scale rates of the frame is one of the main factors limiting the determination of geocentric sea level trends today.

Assessing the accuracy of the ITRF origin has been an important geodetic issue for demanding scientific applications (Dong et al. 2003). Interesting studies involved the use of independent results with absolute gravity data (Teferle et al. 2009; Plag et al. 2007) or a kinematic model of the Earth's crust in order to constrain or discuss the origin of the terrestrial reference frame. In particular, Argus (2007) tried to constrain the origin of the terrestrial reference frame using post-glacial rebound models. The author fitted a translation rate between a kinematic model, composed of a GIA vertical velocity model and Euler pole models for horizontal velocities, and a geodetic velocity field. He found an 'ideal' terrestrial reference frame origin that was reported to be the Centre of Mass of the solid Earth (CE), which was distinct from the ITRF2005 origin by -1.2 ± 0.5 mm/year on the Z component. A similar study carried out by Kogan and Steblov (2008) found a difference of -2.5 ± 0.2 mm/year for quite a similar origin, that they called the Centre of Plate rotation (CP), using only a horizontal velocity field and a Euler pole model. However, the same study carried out by Argus (2007) produced an origin which departed by no more than 0.7 mm/year in norm to what was obtained with the vertical component only. Such studies are part of an effort to find alternative ways to evaluate the ITRF origin. In this paper we present a new approach based on long-term GPS-corrected tide gauge records (over 60 years). Its application is illustrated with valuable results despite the limited set of available long-term records.

To our knowledge the effects of uncertainty in the reference frame datum on GPS-corrected tide gauge records have not been documented in detail although they have been mentioned in the literature (Blewitt et al. 2010). These effects were quantitatively assessed only on altimetry-based estimates of regional and global sea level rise (Morel and Willis 2005; Beckley et al. 2007; Lemoine et al. 2010). In this paper we thus carry out a similar exercise to investigate the sensitivity of estimates of global sea-level rise from GPS-corrected tide gauge records to uncertainties in the ITRF datum. Section 2 formulates the method used to calculate the average rate of global sea-level rise, and presents the data set used in this study. The relationship between vertical velocities and reference frame is then discussed in Sect. 3, outlining how errors in the physical parameters of the reference frame (origin and scale) spread into the vertical velocities. A useful transfer function was derived to assess the sensitivity of estimates of global sea-level rise to uncertainties in the reference frame datum. By considering forward calculations, in Sect. 4, we explore to what extent the exercise could be inverted, providing an evaluation of the reference frame origin. The results and implications are discussed in Sect. 5. Finally, the main conclusions as well as the open questions are summarized in Sect. 6.

2 Calculation of the rate of global sea-level change

We summarize here the approach used by Wöppelmann et al. (2007, 2009), which was adapted from Douglas (2001). The average rate of global sea-level rise for the past century can be calculated in three steps from a set of tide gauge records complying with given criteria of duration, completeness and co-location with a continuous GPS station. These criteria will be reviewed below.

In the first step, the rates of relative sea level change from the tide gauge records are corrected for land motion using the vertical velocities of the co-located GPS stations, yielding individual geocentric sea-level trends for each tide gauge site in the reference frame used to express the GPS velocities. In the second step, regionally averaged rates of sea-level change are computed by grouping the sites into regions according to the apparent spatial correlations of the tide gauge series at decadal and longer periods (Douglas 2001). At this stage, the application of a weighting is suggested to account for the precision of the individual geocentric sea level trends as each trend is supposed to undergo the same (regional) sea-level change. The third step consists in averaging the regional sea-level trends, and subsequently yields an estimate of the average rate of global sea-level change. The procedure can be formulated as follows:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{n_i} p_{i,j}\cdot\left[v_{i,j}^{tg} + v_{i,j}^{up}\right] \tag{1}$$

where dS/dt is the average rate of global sea-level change; N is the number of regions; n_i is the number of stations in a region i; $v_{i,j}^{tg}$ and $v_{i,j}^{up}$ are the rate of the tide gauge record and GPS station height velocity at site j within the region i, respectively; and $p_{i,j}$ is the weighting of the geocentric sea level trend j within the region i such that:

$$\sum_{j=1}^{n_i} p_{i,j} = 1 \tag{2}$$

If no specific weighting is adopted then $p_{i,j} = 1/n_i$.

Wöppelmann et al. (2007, 2009) considered tide gauge records containing more than 85% of valid data within minimum time spans of 60 years from the Permanent Service for Mean Sea Level (PSMSL) 'Revised Local Reference (RLR)' dataset. The RLR dataset is particularly appropriate for examining trends in long term sea-level studies as its records were checked and corrected for local datum changes (Woodworth and Player 2003). The geocentric vertical velocities come from the solution proposed by the University of La Rochelle (ULR) Analysis Centre published in (Wöppelmann et al. 2009).

For the sake of completeness the main characteristics of the ULR analysis strategy are summarized here. The GPS observations from a global network of 227 stations were analyzed using the same parameterization and observation modeling over the whole observation period from January 1997 to November 2006. A double-difference approach was used, estimating station coordinates, satellite orbits, earth orientation parameters and zenith tropospheric delay parameters as a piecewise linear model with nodes every 2 h. An important feature was the use of absolute antenna phase center corrections for satellites and receivers. Daily GPS observation files were grouped into five sub-networks with up to 50 stations each. To ensure the optimal estimation of satellite orbital parameters, as well as the alignment to the ITRF2005 reference frame, the sub-network stations were globally distributed. Loosely constrained sub-network solutions were produced using a priori site coordinates in the ITRF2005 reference frame, a priori orbits from the IGS, and a priori Earth orientation parameters from the IERS Bulletin B. The sub-network solutions were combined and aligned to ITRF2005, generating global daily solutions (and then weekly solutions from the daily solutions). Further details on the ULR processing strategy are available in the electronic supplement of (Wöppelmann et al. 2009). The GPS technique is theoretically sensitive to the CM but it is outside the scope of this paper to evaluate this information. Wöppelmann et al. (2009) chose to refer their velocity field with respect to the origin and scale of ITRF2005. They used a minimum constraint equation that relies on the similarity transformation described in Eq. 1 of Appendix A for such a purpose.

Figure 1 shows in color the distribution of the 27 stations used by Wöppelmann et al. (2009) that comply with our criteria. GPS antennas less than about 20 km from the tide gauge were considered to be co-located i.e., the relative movements between the GPS antenna and the tide gauge were assumed to be negligible. Bevis et al. (2002) stressed that the critical issue in GPS monitoring of tide gauges is the relative local stability, not the distance. Vertical motion could indeed be significantly different at just a few metres distance on an unstable pier. In contrast, GPS and tide gauge stations may be separated by several kilometres as long as the bedrock upon which the instruments are settled undergoes the same vertical motion. In any case, a distance criterion with a subsequent working hypothesis was necessitated here by a lack of levelling data between the GPS antenna and the tide gauge benchmark. The co-located GPS stations were also required to show vertical velocities with standard errors several times smaller than the expected rates of sea-level rise of 1–2 mm/year, hence supporting the exercise of correcting the tide gauge rates with GPS vertical velocities of comparable standard errors to those of the tide gauges. Note that the uncertainties of the GPS vertical velocities were computed from maximum likelihood estimates of correlated noise variances (Williams 2008). These are known to be more realistic than GPS velocity formal errors obtained under white noise assumption (Langbein and Johnson 1997; Zhang et al. 1997). The colors of the stations in Fig. 1 highlight their morphological grouping based on the spatial correlations of the tide gauge series at low frequencies. Table 1 shows the numerical results, which were used in our study as input data. The Fernandina record, Gulf of Mexico, was definitively discarded as its anomalous geocentric sea-level rate was

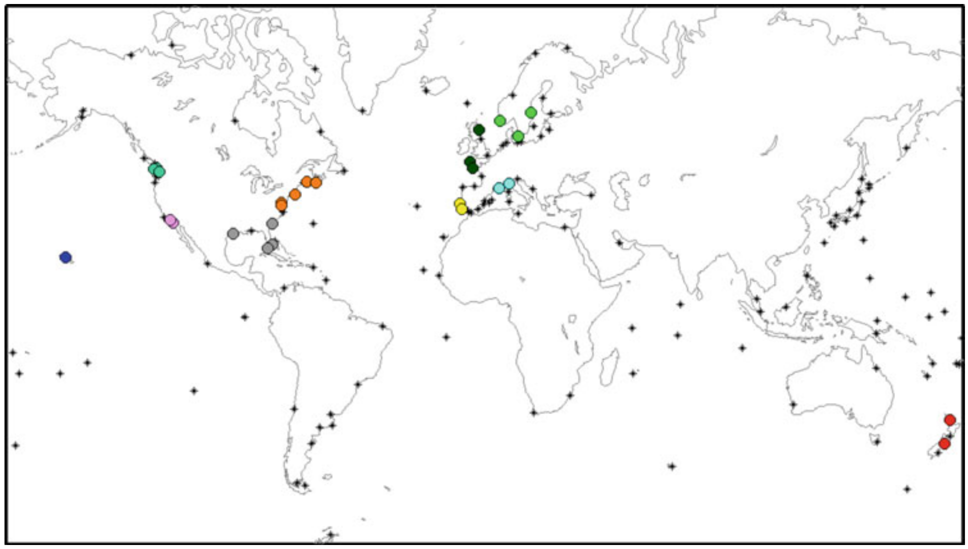

Fig. 1 Distribution of the tide gauge co-located GPS stations processed by Wöppelmann et al. (2009). The *circles* represent the stations complying with given criteria (see text) required in the calculation of global sea-level rise. The *colors* highlight their morphological grouping based on spatial correlations of tide gauge series at low frequencies. *Black crosses* indicate the existing GPS co-located tide gauge stations that do not yet meet the selection criteria

recently discovered to come from a mistaken co-location with a GPS station several hundreds of kilometers away.

Tide gauge records have an averaged time span of 80 ± 17 years compared to 8 ± 2 years for GPS records. An important but necessary assumption worth emphasizing here is that the land motions are assumed to be constant over the time span of the tide gauge records and are faithfully reflected by the GPS velocities (further discussed in Sect. 5). No significant inter-annual motions were detected in the GPS station position time series. However, low frequency signals were identified as background flicker noise. They are accounted for in the evaluation of velocity uncertainty. Contrary to Wöppelmann et al. (2009), weighting was used to compute the regional rates of sea-level change in ITRF2005. Weights were computed from the uncertainties in geocentric sea level trends, as shown in the sixth column of Table 1. Table 1 also provides the geocentric sea-level rates of each group of stations using this kind of weighting. Finally, the globally averaged sea-level rise computed using Eq. 1 was 1.59 ± 0.09 mm/year, which agrees with the value obtained by Wöppelmann et al. (2009) without the weighting scheme and covariance propagation (1.61 ± 0.19 mm/year).

3 Reference frame and sea-level trends

3.1 Vertical velocities

Sea-level, and subsequently 'sea-level change', can be ambiguous as it can be given relative to a benchmark on land, or relative to a conventional terrestrial reference frame. In the first instance, the attribute 'relative' is usually added explicitly to designate the sea-level as observed by tide gauges at coastal locations (denoted 'relative sea-level', RSL henceforth), stressing the fact that tide gauge-determined rates of sea level change reflect land motion in addition to ocean mass and volume changes. However, RSL is a physical concept which can be defined as the distance between the sea surface and the surface of the solid Earth, and is accessible through direct observation. In contrast, geocentric sea-level change and land motion are more abstract concepts, which require careful definition in their scientific application (Blewitt 2004). They are reference frame-dependent quantities, which are therefore sensitive to the definition of the frame characteristics (origin and scale). Consequently, differences in velocity estimates may simply reflect uncertainty in the frame definition or inability to obtain the adopted frame in practice.

From Eq. A1 of Appendix A it was possible to derive the impact of an uncertainty in the origin and scale of the reference frame on the vertical velocity $v_{i,j}^{up}$ of a station j within a region i. Origin and scale uncertainties were modeled by translation and scale rate errors $\Delta\dot{\mathbf{T}}$ and $\Delta\dot{d}$. $\Delta\dot{d}$ is a scale factor rate error given here in millimeter per year for the sake of unit consistency. It can be computed by multiplying the original scale factor by the mean radius of the Earth. This approximation, computed on our network, leads to a maximal error of 8×10^{-3} mm/year with a scale factor rate of 1 ppb/year. By substituting $\dot{\mathbf{T}}$ with $\Delta\dot{\mathbf{T}}$ and $\dot{s} \cdot \mathbf{X}$ with $\Delta\dot{d}$ in Eq. A1 of Appendix A, we can deduce that the vertical velocity $v_{i,j}^{up}$ is affected by the two origin and scale error terms:

$$\Delta v_{i,j}^{up}(\Delta\dot{d}, \Delta\dot{\mathbf{T}}) = \Delta\dot{d} + \mathbf{G}(\lambda_{ij}, \phi_{ij}) \cdot \Delta\dot{\mathbf{T}} \tag{3}$$

where $(\lambda_{ij}, \phi_{ij})$ are the longitude and latitude of station i, j and $\mathbf{G}$ the rotation matrix described in Appendix A. Equation 3 shows that any scale rate error will totally transfer to the vertical velocity, and subsequently to the geocentric sea level trend obtained from a GPS-corrected tide gauge record. The translation effect depends closely on the geographic location

Table 1 Adapted from Wöppelmann et al. (2009). Rates of relative sea-level, GPS vertical velocity in ITRF2005, and geocentric sea-level change at tide gauge stations complying with given criteria (see text)

Groups of stations	Tide gauges (TG)		GPS w.r.t. ITRF2005		TG+GPS	Group
	Span (year)	Trend (mm/year)	Span (year)	Trend (mm/year)	Trend (mm/year)	Trend (mm/year)
North Sea Eng. Channel						
Aberdeen I+II	103	0.58 ± 0.10	8.2	0.67 ± 0.22	1.25 ± 0.24	1.32 ± 0.18
Newlyn	87	1.69 ± 0.11	8.1	-0.21 ± 0.27	1.48 ± 0.29	
Brest	83	1.40 ± 0.05	8.0	-0.54 ± 0.77	0.86 ± 0.77	
Atlantic						
Cascais	97	1.22 ± 0.10	8.1	0.12 ± 0.19	1.34 ± 0.21	1.32 ± 0.18
Lagos	61	1.35 ± 0.18	6.6	-0.10 ± 0.29	1.25 ± 0.34	
Mediterranean						
Marseille	105	1.27 ± 0.09	8.3	0.82 ± 0.37	2.09 ± 0.38	1.92 ± 0.35
Genova	78	1.20 ± 0.07	8.3	-0.16 ± 0.85	1.04 ± 0.85	
New Zealand						
Auckland II	85	1.30 ± 0.13	5.3	-0.87 ± 0.48	0.43 ± 0.50	1.12 ± 0.29
Port Lyttelton	101	2.08 ± 0.11	7.0	-0.59 ± 0.35	1.49 ± 0.37	
Pacific						
Honolulu	99	1.46 ± 0.13	8.6	-0.15 ± 0.36	1.31 ± 0.38	1.31 ± 0.38
SW North America						
La Jolla	72	2.11 ± 0.16	9.8	-0.38 ± 0.62	1.73 ± 0.64	1.01 ± 0.39
Los Angeles	78	0.86 ± 0.15	7.9	-0.30 ± 0.48	0.56 ± 0.50	
SE North America						
Charleston I	82	3.23 ± 0.16	6.9	-1.31 ± 0.44	1.92 ± 0.47	1.71 ± 0.25
Galveston II	94	6.47 ± 0.17	5.9	-5.89 ± 0.61	0.58 ± 0.63	
Miami Beach	45	2.29 ± 0.26	6.7	0.46 ± 0.61	2.75 ± 0.66	
Key West	90	2.23 ± 0.10	9.4	-0.59 ± 0.38	1.64 ± 0.39	
NE North America						
Eastport	63	2.07 ± 0.16	8.1	2.07 ± 0.87	4.14 ± 0.88	2.70 ± 0.22
Newport	70	2.48 ± 0.14	7.3	0.42 ± 0.37	2.9 ± 0.40	
Halifax	77	3.29 ± 0.11	3.9	-0.72 ± 0.31	2.57 ± 0.33	
Annapolis	70	3.46 ± 0.17	8.9	0.69 ± 0.94	4.15 ± 0.96	
Solomon's ISL.	62	3.36 ± 0.19	9.8	-2.43 ± 0.69	0.93 ± 0.72	
Northern Europe						
Stavanger	63	0.27 ± 0.17	6.0	2.68 ± 0.82	2.95 ± 0.84	1.28 ± 0.22
Kobenhavn	101	0.32 ± 0.12	3.9	0.97 ± 0.35	1.29 ± 0.37	
Nedre Gavle	90	-6.05 ± 0.23	7.7	7.12 ± 0.19	1.07 ± 0.30	
NW North America						
Victoria	86	1.10 ± 0.15	9.8	1.20 ± 0.23	2.30 ± 0.27	2.26 ± 0.20
Neah Bay	65	-1.59 ± 0.22	8.8	3.82 ± 0.69	2.23 ± 0.72	
Seattle	104	2.06 ± 0.11	8.8	0.14 ± 0.31	2.20 ± 0.33	

Standard errors of the geocentric sea level rates were computed assuming that GPS and tide gauge data are independent. The last column gives the geocentric regional sea-level rates of each group of stations using a weighted average

of the station but it is worth noting that a Z translation rate mostly affects the vertical velocities at high latitude (see Eqs. 3 and A2). We could anticipate a compensation of the translation error when determining globally averaged sea level rise from an evenly distributed tide gauge network. However, due to the non-uniform distribution of the ocean, this term will never completely average out. Moreover, tide gauges with long records and useful geodetic information on vertical land motion are not well distributed (Fig. 1). Section 3.2 focuses on the transfer function from reference frame datum errors

Table 2 Propagation factors for reference frame error in the global sea-level rise estimate to be multiplied by origin and scale errors as a function of the network distribution of stations

Network of stations extracted from Wöppelmann et al. (2009)	$\Delta\dot{T}_x$	$\Delta\dot{T}_y$	$\Delta\dot{T}_z$	$\Delta\dot{d}$
Selected set of 27 stations grouped into 10 morphological regions (see text)	0.060	−0.308	0.509	1
Complete network (crosses in Fig. 1, no grouping)	0.043	−0.061	0.301	1
1 per 10 × 10 deg. cell set of stations	0.001	−0.019	0.093	1

to global sea level rise estimates from tide gauge and GPS data.

3.2 Impact on the global sea-level rise estimates

Using Eqs. 1 and 3, we derived Eq. 4 which could be used to evaluate the propagation of any scale and origin drift error to the estimate of global sea level rise.

$$\Delta\frac{\mathrm{d}S}{\mathrm{d}t}(\Delta\dot{\mathbf{T}},\Delta\dot{d})=\Delta\dot{d}+\left[\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{n_i}p_{i,j}\mathbf{G}(\lambda_{ij},\phi_{ij})\right]\cdot\Delta\dot{\mathbf{T}} \tag{4}$$

This transfer function applies to errors in the underlying reference frame datum of the velocity field. As previously noted, an error in the drift of the reference frame scale will affect the estimate of the global rate of sea-level change by the same amount. It was confirmed that the effect of an origin error is heavily dependent on the geographical distribution of the stations. The transfer functions associated with origin errors were evaluated for various network configurations using Eq. 4. The results are provided in Table 2. The largest error propagation factor was found to be 51% for the network of 27 selected stations grouped in 10 regions used by Wöppelmann et al. (2009), coming from errors in the Z component of the origin drift. This is understandable since most of these stations are located in the Northern hemisphere at mid-latitudes (Fig. 1). The Y component of the origin drift followed, with a propagation factor of about 30%. The lowest impact, with a factor of less than 10%, was obtained with the X component.

The transfer function could further be used to re-assess the contribution of the reference frame error to the error budget of the estimate of global sea level change. For instance, the global sea level rise re-estimated from Wöppelmann et al. (2009) using the weighting scheme suggested in Sect. 2 was 1.59 ± 0.09 mm/year in ITRF2005. We took an uncertainty of 0.5 mm/year for the scale rate of the terrestrial reference frame and of 1.0 mm/year for the rate of the origin's Z component, which is believed to be the current determination of origin and scale by space geodesy (Z. Altamimi, personal communication). In practice, errors in reference frame origin and scale may be correlated since SLR and VLBI networks, which define ITRF2005 origin and scale, are unevenly distributed. As an indication, and to further support the need to reappraise the error bar in the rate of global sea level change by considering reference frame uncertainties, we assumed a correlation of −0.10 between Z translation drift and scale drift (see Sect. 5). By propagating the variance using Eq. 4 with the above values, we found a resulting error of 0.69 mm/year. Thus, the estimate of global sea-level rise would be revised to 1.59 ± 0.69 mm/year.

We assumed here that the reference frame errors (which are rather systematic) are independent from the GPS vertical velocity errors (which were derived from the non-linear variations in the time series). Nevertheless, this underlines the fact that the current reference frame errors are preponderant in the error budget, and should therefore be carefully assessed to provide a realistic uncertainty estimate for global sea-level rise. It should also be noted that the estimates of global sea-level rise from Wöppelmann et al. (2007, 2009) reviewed in the introduction fit within the new error bars. The transfer function of Eq. 4 should also be used to compute the globally averaged sea-level rise in any other terrestrial reference frame, provided the transformation parameters are known. For example, when the next realization of the ITRS is released (ITRF2008), transformation parameters will be delivered with respect to ITRF2005. If non-zero transformation parameters are published, they could be used to evaluate global sea-level rise with respect to the new ITRF from the Wöppelmann et al. (2009) estimate.

The geographical distribution of the selected set of long-term tide gauge records co-located with GPS stations was sparse and highly uneven. So, we further investigated the network geometry effect using improved distributions of stations based on existing GPS co-located tide gauge stations that will hopefully meet the selection criteria of Sect. 2 in the near future, as new data is recorded and made available (see black crosses in Fig. 1). The transfer functions of the reference frame errors to the global averaged estimates of sea level rise are given in Table 2 for two simulations. First, every station was assumed to be part of a distinct region. The transfer function on the Z component was still quite high, with a value of 30%. If well distributed regional groups were constituted, or a well distributed sub-network was extracted, it was possible to mitigate that value to less than 10%. We obtained such a result by selecting one tide gauge station per 10×10 degree cell (Table 2), for instance. Merrifield et al. (2009b) have recently suggested a different averaging process based on latitude band grouping. If GPS corrections had been used, Eq. 3 would have been adequate to evaluate the impact of the reference frame error. But due to the weighting configuration, a Z origin error would have had a limited effect. In conclusion, our simulations suggest that a well distributed network significantly attenuates the impact of error in the reference frame origin on the estimate of global sea level rise from GPS-corrected tide gauge records. However regional errors are still significant but they can always be evaluated using Eq. 3.

4 Sea level constraints on the reference frame origin

4.1 Method

Tide gauge records reflect both sea level rise and vertical land motion. If the geocentric sea level rise is known, vertical land motion velocities can be easily derived from the tide gauge records. This is the basic reasoning used by Cazenave et al. (1999); Nerem and Mitchum (2002); Kuo et al. (2004). They assumed that altimetry is a perfect sensor of geocentric sea level rise. However, the geocentric sea level rise and in particular its spatial variability are not accurately known on the century time scale that is considered in our study. The question is posed here as to whether the geographical scatter observed in the regional GPS-corrected tide gauge rates (Wöppelmann et al. 2009) could be used to constrain the terrestrial reference frame parameters. The idea of searching for a reference frame in which the sea-level rise spatial variability is minimized is developed below.

We considered the empirical variance of the regional differences in sea-level rise with respect to the global average as a function of the origin and scale rate errors of the ITRF2005 frame. The expression of this function is given in Appendix B, Eq. A3. We searched for the translation and scale rate parameters that minimized this quantity based on the network of stations and data provided in the last column of Table 1. Unfortunately, the considered function was not dependent on the scale rate error (see Eq. A4). Nevertheless, the origin of the reference frame could be constrained. Indeed, the independence of the scale rate error was unfortunate since the associated transfer function was found to be the largest (100%, Sect. 3), but the contribution of origin error could be as high. It could also be regarded as an advantage since scale and origin errors could be separated using this method. The translation that minimizes the variance of the regional sea-level rates is the result of a system of three equations and three unknowns. It could be obtained by nullifying the derivatives of the empirical variance with respect to the translation parameters. The system of equations is given in Appendix B, Eqs. A5–A7. In addition, as the solution has a linear dependence on the tide gauge and GPS velocities, it is easy to derive the covariance matrix of the translations.

The implicit hypothesis that defines our 'ideal' reference frame is that the spatial variability in sea level rise on the century time scale is expected to be smaller than the contribution of reference frame error. However, there are several geophysical processes that may actually cause non-uniform secular sea level changes. GIA is one of them since it is known to cause major geoid changes, including the contribution of rotational feedback (Peltier 1999). Sea-level rise is predicted to vary spatially as a result of the redistribution of glacial ice and melt-water from the continents into the oceans (Conrad and Hager 1997; Mitrovica et al. 2001). The predicted spatial variations are generally long wavelength (>1,000 km), and further legitimated the regional grouping and weighting scheme proposed in Sect. 2. However, it should be noted that no clear evidence of fingerprints of glacial melting has been found in long tide gauge records so far (Douglas 2008), either because other signals in tide gauge records mask them, or because the net effect of the different reservoir partly compensates. In addition, sea-level changes due to ocean thermal expansion have their own spatial pattern depending on where the heat content is increased or decreased (Ishii et al. 2006), further complicating the global geometry of fingerprints and their detection.

In the present study, we did not introduce geophysical models of the above contributions into the estimation of the reference frame bias as not all the contributions are accurately known yet, especially the steric contribution. However, we evaluated the effect of GIA as it was considered to be the dominant effect that may bias our estimation. We used GIA predictions from Peltier (2004) that were available at the Special Bureau for Loading (http://www.sbl.statkart.no/projects/pgs/). Two different sets of GIA predictions were considered. They differ by the viscosity profile used to compute the load Love numbers, but both were driven by the same ice history (ICE-5G) (Peltier 2004). The geocentric sea level rates that can be computed by summing the GIA predicted rates of relative sea level change and the GIA predicted radial displacements refer to the CE frame (Farrell 1972). As the geocentric sea level is an equipotential of the gravity field, we expressed the GIA-derived geocentric sea level rates in the CM frame by removing the degree 1 term of the gravity field potential (L. Métivier, personal communication). We could then compute an origin rate error from the GIA-derived geocentric sea level rates in the CM frame. Table 3(a) shows the estimated translations computed using the network of 27 stations of Fig. 1, using the two Earth models ("VM2 CM

Table 3 Estimated translation resulting from the minimization approach described in the text using the models, (a), and data of Table 1 for various networks based on the selected set of 27 stations grouped into morphological regions, (b) including all stations, (c) removing certain stations for sensitivity analysis

	$\Delta\dot{T}_x$ (mm/year)	$\Delta\dot{T}_y$ (mm/year)	$\Delta\dot{T}_z$ (mm/year)
(a) Synthetic geocentric sea level from GIA models			
VM2 CM min T	0.27 ± 0.06	-1.34 ± 0.06	-0.48 ± 0.08
VM2 CM min Tz	0.00	0.00	-0.04 ± 0.05
VM4 CM min T	0.28 ± 0.06	-1.15 ± 0.06	-0.42 ± 0.08
VM4 CM min Tz	0.00	0.00	0.01 ± 0.05
(b) Real data			
ITRF2005 min T	-0.23 ± 0.25	0.63 ± 0.28	-0.14 ± 0.31
ITRF2005 min Tz	0.00	0.00	-0.44 ± 0.22
(c) Sensitivity analysis			
Southern hemisphere stations removed	-0.16 ± 0.23	0.92 ± 0.36	-1.47 ± 0.77
	0.00	0.00	-0.64 ± 0.67
Fennoscandian stations removed	-0.17 ± 0.27	0.85 ± 0.41	-0.25 ± 0.38
	0.00	0.00	-0.55 ± 0.23

0.00 is given when the parameter is set at 0

min T" and "VM4 CM min T" rows). We assumed that the uncertainty of the GIA geocentric sea level predictions was 0.1 mm/year in order to supply our formula with standard errors (Appendix B).

Both GIA models yielded similar conclusions. The estimated translation rate due to GIA was significant, especially the Y term, which reached -1.3 mm/year, and the translation rate components were correlated. This showed that such an approach was not possible in practice with the network used in this study. We further explored the approach for solving only the Z translation rate shift, in this way canceling the effect of high correlations between the translation vector components. By doing so, we assumed zero rate shifts in the X and Y components. Indeed, the configuration of satellite orbits with respect to the rotating Earth tends to limit the errors on the equatorial components of the reference frame origin (Morel and Willis 2005). The estimated Z origin error terms were smaller than 0.05 mm/year, showing that GIA does not significantly impact our estimate if only the Z component is considered. We also found a regional variability of the geocentric sea level rise of 0.49 mm/year due to GIA. The tests further suggested that our morphological grouping effectively mitigated the spatial correlations expected from GIA. As a result, the assumption was introduced that the geocentric sea level rates computed for each group were not spatially correlated. We also assumed that neglecting the effects of melting ice and temperature change over the last century would not significantly impact the estimation of translation rate error due to our network distribution.

4.2 Results

We solved the Z translation rate error with respect to ITRF2005 using the equations in Appendix B and our network of 27 stations. Figure 3 shows the rate of global sea-level change and regional sea level rise scatter as a function of the Z translation rate shift with respect to ITRF2005 origin. It illustrates how the regional sea level rise scatter is dependent on the reference frame origin. The resulting error in the time evolution of the origin is supplied in Table 3(b) and was -0.44 ± 0.22 mm/year along the Z component (line "ITRF2005 min Tz"). We also supplied the estimation of the translation rate when the three components are estimated simultaneously under the name "ITRF2005 min T". A χ^2 statistical test was performed to investigate the significance of the resulting translation rate shift with respect to a null translation rate shift. The test showed that the estimated translation rate shift was not significant at the 95% level. The largest translation rate shift was found on the Y component with a value of 0.63 ± 0.28 mm/year. This value could be related to the large trend of the geocentric sea-level rate in the North East group of North America (Fig. 2), which tends to be compensated by a Y translation. Figure 2 shows the individual rates of geocentric sea-level change with respect to ITRF2005 as a function of latitude (the colors are consistent with those used in the map shown on Fig. 1).

The vector of the adjusted reference frame translation was found to be sensitive to the network of sea-level stations used to determine it. Although our network is quite sparse, we carried out a sensitivity analysis by removing some of the

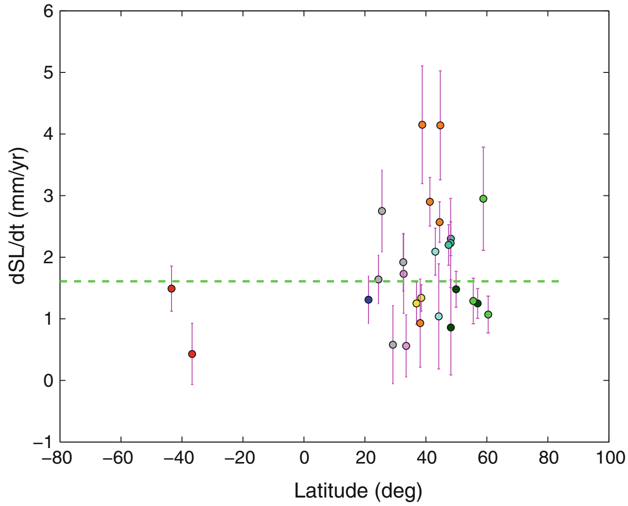

Fig. 2 Individual geocentric sea level trends in ITRF2005 from Table 1 as a function of station latitude. The *colours* correspond to the groups of sites in Fig. 1

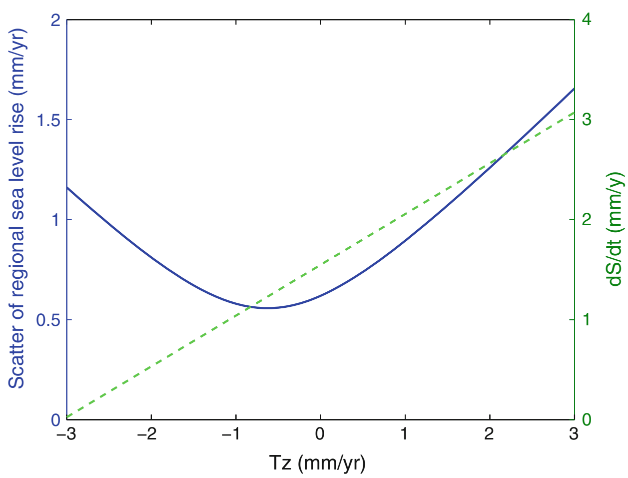

Fig. 3 Rate of global sea-level change (*dashed-line*, *right-hand axis*) and scatter of regional sea-level rise (*plain-line*, *left-hand axis*) as a function of a Z translation rate shift with respect to the ITRF2005 origin

regions of stations and investigating how much the resulting translation was affected (see Table 3(c)). Removing stations in the Southern hemisphere led to a relatively large translation rate vector of $[-0.16 \pm 0.23;\ 0.92 \pm 0.36;\ -1.47 \pm 0.77]$ mm/year which demonstrated the importance of having a well distributed network. However, estimating only the Z component yielded a more stable result of -0.64 ± 0.67 mm/year. By removing the group of Fennoscandian stations in Northern Europe, we came up with a Z-translation rate estimate of -0.55 ± 0.23 mm/year. The tests showed that the translation rate estimate proved to be quite stable if only the Z component of the origin was estimated, which was probably related to our network configuration. This was considered rather positive as the Z component of the origin is usually the component where error is the most probable. We will now discuss the results obtained when the X and Y components of the error in the origin rate were set at zero since the results in that case were shown to be more reliable by the GIA data tests and the sensitivity analysis.

5 Discussion

We introduced in Sect. 3.2 the effect of a possible correlation between errors in reference frame origin rate and scale rate on the error of the averaged rate estimate of global sea level change. Indeed, in the ITRF2005 production process, the SLR origin is transferred to GPS by means of a 14-parameter transformation and the same can be said for the scale information from VLBI. As the co-location of the GPS network with these two techniques is unevenly distributed, the GPS translation and scale rates with respect to the ITRF2005 solution should be correlated. From the ITRF2005 solution covariance matrix (Z. Altamimi, personal communication), we evaluated the correlation between the GPS Z translation rate and the GPS scale rate, with respect to the ITRF2005 solution, to be -0.10 with uncertainties at the level of 0.1 mm/year. These values reflect how GPS frame is tied to SLR origin and VLBI scale in the ITRF2005 solution but also which portion of a possible error in the time evolution of the origin could be interpreted as scale rate error. If we assume that the error that we estimated in the previous section reflects a misalignment of the GPS frame with respect to the SLR and VLBI frames, the correlation is sufficiently small to allow us to discuss errors in the origin independently from the scale factor. If the correlation had been larger and our origin rate error estimated in the previous section better determined, it would have been possible to solve the scale rate error a posteriori from our translation rate error. Indeed, assuming that the covariance matrix between the Z translation rate error and the scale rate error is known, we could have estimated the scale rate error using least squares collocation, for example (Moritz 1989). As the correlation is small, we evaluated the global sea level rise in the ideal reference frame without considering any scale effect. The rate of global sea level change was found to be 1.30 ± 0.15 mm/year in the ideal terrestrial reference frame (Table 4), which is 0.29 mm/year smaller than in ITRF2005. The uncertainty of the global sea-level rise expressed in the shifted reference frame was evaluated by propagating the uncertainty in the origin of this reference frame and accounting for its covariance terms with the tide gauge and GPS data.

Table 4 summarizes the scatter of the regional sea-level rates (standard deviation) and the corresponding rate of global sea-level change for various reference frames, either given (like ITRF2000 and ITRF2005), or resulting from the minimization approach described above using the data of Table 1.

Table 4 Scatter of the regional sea-level rates (standard deviation) and corresponding rate of global sea-level change for various reference frames, either given (ITRF2000 and ITRF2005), or resulting from the minimization approach described in the text using the data of Table 1

Strategy and reference frame parameters (if it applies)					Scatter (mm/year)	dS/dt (mm/year)
No land motion correction					1.37	1.36 ± 0.03[a]
	$\Delta\dot{T}_x^{b}$ (mm/year)	$\Delta\dot{T}_y^{b}$ (mm/year)	$\Delta\dot{T}_z^{b}$ (mm/year)	$\Delta\dot{d}^{b}$ (ppb /year)		
GPS velocity corrections: ITRS realizations						
ITRF2005	–	–	–	–	0.55	1.59 ± 0.09
ITRF2000	0.00	0.00	−1.80	0.08	0.78	1.19 ± 0.09
GPS velocity corrections: frame derived from minimum variance of regional sea-level rates						
ITRF2005 min Tz	0.00	0.00	-0.44 ± 0.22	0.00	0.51	1.30 ± 0.15[c]

As a guideline, the case where no land motion correction was applied is included. The uncertainties attached to the estimates of global sea-level change were obtained from the tide gauge and GPS velocity uncertainties using covariance propagation

[a] This uncertainty only reflects formal errors of the tide gauge record rates

[b] Transformation parameters are supplied with respect to ITRF2005

[c] The uncertainty takes into account the uncertainty in the origin definition and covariance terms with geocentric sea level trends

As a guideline, the case where no land motion correction was applied is included in Table 4. The smallest scatter of long-term regional sea-level change rates was found to be 0.51 mm/year in the sea-level constrained reference frame where only the Z component was adjusted, which is a lower scatter compared to 0.78 mm/year in ITRF2000, but quite close to 0.55 mm/year in ITRF2005 (Table 4). The median value of the regional geocentric sea-level rate error was evaluated at 0.24 mm/year, which could be considered as additional evidence showing that the variability of the geocentric sea level trend was real. Another source of error in our estimates could arise from the assumption of stationarity in the GPS velocities. If a GPS velocity was different over its observing period compared to the longer tide gauge period, the difference would partly map onto the reference frame errors. However, the assumption is supported by the very small scatter of the acceleration term in tide gauge records longer than 50–60 years, as was pointed out by (Douglas 2001, Figure 3.16, pp. 61), suggesting that vertical land motion rates are nearly constant and essentially linear on the 100-year time span and tide gauge sites considered here.

Assessing the accuracy of the ITRF origin has for long been an important geodetic issue. Altamimi et al. (2008) provided a conservative evaluation of the accuracy of the current origin and scale drift at the level of 1.8 mm/year and 0.1 ppb/year, respectively, based on a straightforward comparison between the two latest reference frames, ITRF2000 and ITRF2005. However, this evaluation of origin uncertainty might be pessimistic as the SLR solutions that were used to define the ITRF2000 origin pointed to an agreement at the level of 0.5–1.0 mm/year on the Z component (Altamimi et al. 2002). That was the reason why we previously used an uncertainty of 0.5 mm/year in the terrestrial reference frame scale and 1.0 mm/year in the Z component of the origin (Z. Altamimi, personal communication). Here we provide an evaluation of ITRF frame origins that could complete previous studies (Argus 2007; Kogan and Steblov 2008; Teferle et al. 2009). Based on the new constraints developed in this article and the sea-level data from the GPS co-located tide gauge network given in Table 1, we found a reference frame origin that falls between ITRF2005 and ITRF2000, at 1.36 ± 0.22 mm/year from ITRF2000 along its Z component, and at -0.44 ± 0.22 mm/year from ITRF2005. Our result yielded a distinct origin from that of Argus (2007) and Kogan and Steblov (2008) but was in agreement with the expectations of a reference frame (ITRF2005) that was built with more recent geodetic data and more advanced modeling than the previous one (ITRF2000).

In the sea-level adjusted reference frame, which differs from ITRF2005 by -0.44 ± 0.22 mm/year along the Z component, the global sea level rise was estimated to be 1.30 ± 0.15 mm/year. From the range of values given in Table 4 a bound on the global sea-level rise estimates of 1.2–1.6 mm/year could be inferred, depending on the adopted reference frame. These values fall well within the range of 1–2 mm/year widely quoted for the rise over the last 100 years (Bindoff et al. 2007). However, it suggests that previous estimates (Holgate 2007; Jevrejeva et al. 2008; Wöppelmann et al. 2009) represent an upper bound, as well as a significant departure from altimetry-based estimates of 3.3 ± 0.4 mm/year over the recent period from 1993 to 2007.

6 Conclusion

By studying the relation between tide gauge-determined global sea-level rise and terrestrial reference frame definition, we addressed two important planetary-scale parameters of paramount interest for climate change and geodesy. Scientific interpretation of global sea-level rise should be mindful of the uncertainties surrounding reference frames in which the sea-level change is estimated. In this paper, we established a transfer function which could be used to assess the impact of frame origin and scale uncertainties on the global sea-level rise estimate from GPS-corrected tide gauge records. This transfer function assumed that the terrestrial reference frame is implicitly inherent to the GPS velocity field. It could not be used to account for the errors in the a priori terrestrial reference frame used for the processing of GPS data. In contrast to satellite altimetry-based determinations (Morel and Willis 2005), any error in the scale rate of the reference frame that was used to express the land motion of the tide gauges mapped entirely into the global sea level rise estimation. The effect of any error in the origin rate proved to be dependent on the geometry of the tide gauge network. The largest impact factor came from errors in the Z component of the origin if a network of carefully selected tide gauges was used. However, this factor decreased rapidly if existing stations that had not yet met the selection criteria were included, especially in the Southern hemisphere.

Up to now SLR data have been the only source of information used to realize the origin of the highest standard terrestrial reference frame, given its sensitivity to the center of mass of the whole Earth. We explored a new approach that assessed this accuracy, and might further be developed to provide new constraints on the origin. A reference frame could be found in which the scatter of the regional sea-level rates, computed over 80-year long records on average, was minimum. Its origin is very close to the ITRF2005 origin but significantly different from the ITRF2000 origin. This frame relies on the assumption that the scatter associated with the regional geocentric sea level rise should be minimized in the CM frame. Using model predictions, we showed that the GIA does not affect our estimates if only the Z component of the time evolution of the origin error is adjusted. Unfortunately, our approach could not directly tackle the issue of the reference frame scale. Assuming a given rate of global sea-level rise of 1.8 mm/year, Bouin and Wöppelmann (2010) provided an error estimate of -0.13 ± 0.08 mm/year on the ITRF2005 scale using a different sea-level approach based on an extended network of 70 GPS stations co-located with tide gauges. Their result confirmed the ITRF2005 time scale evolution, but a different value for global sea level rise would have led to a different scale error estimate.

We have shown that tide gauge records of sea-level change contain more information than has been recognized in previous studies, which is useful for both sea-level science or geodesy (Wöppelmann et al. 2006). This constitutes a preliminary application of our procedure in several respects, but it is demonstrative. First, we focused on a restricted set of carefully selected tide gauge and GPS data given by Wöppelmann et al. (2009) but in the future, as data are accumulated and time series lengthen, a larger number of robust GPS-corrected tide gauge records that meet the selection criteria could be incorporated, providing a better geographical coverage. Data archaeology exercises (Woodworth et al. 2005; Testut et al. 2006), in particular in the Southern Hemisphere and former colonial countries, represent an invaluable source of new (rediscovered) long-term sea-level records. In addition, significant progress has recently been made in extending the global network of tide gauges, such as that being developed by the Global Sea Level Observing System (GLOSS, Merrifield et al. (2009a)), in particular to monitor land motion at the tide gauges with continuous GPS stations (Pilot project TIGA, Schöne et al. (2009)). Geocentric sea level trends averaged over a larger network would supply more reliable estimates of globally-averaged rates of sea level change comparable with almost truly global estimates from altimetry (Prandi et al. 2009). Secondly, the procedure could cautiously be extended to satellite altimetry data whose near-global geographic coverage overcomes the tide gauge distribution problem and provides an estimate of geocentric sea level rise. It would also be interesting to examine other data sets, such as predictions of spatial variations in sea-level (fingerprints) due to glacial melting in the major land-based ice reservoirs (Antarctic, Greenland and smaller mountain glaciers and ice sheets), to improve our a priori knowledge of the spatial variability of the geocentric sea level rise.

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Appendix A: Velocities in various reference frames

In the case of terrestrial reference frames derived from space geodesy, the differences of positions and velocities expressed with respect to two different reference frames can be formulated by:

$$\begin{aligned}\mathbf{X}' &= \mathbf{X} + \mathbf{T} + s \cdot \mathbf{X} + \mathbf{R} \cdot \mathbf{X} \\ \mathbf{V}' &= \mathbf{V} + \dot{\mathbf{T}} + \dot{s} \cdot \mathbf{X} + \dot{\mathbf{R}} \cdot \mathbf{X}\end{aligned} \tag{A1}$$

where $(\mathbf{X}, \mathbf{V})$ and $(\mathbf{X}', \mathbf{V}')$ are two sets of coordinates in two distinct reference frames, consisting of the position and velocity vectors of one point in the three-dimensional space. In addition, the relationship consists of 14 so-called transformation parameters which are the scale factor s, the translation vector $\mathbf{T}$, and the antisymmetric rotation matrix $\mathbf{R}$, as well as their respective time derivatives (Dot notation is used henceforth for time derivatives). The relationship is obtained by linearizing a similarity, assuming that the orientation and scale which link terrestrial reference frames derived from space geodesy are small (Altamimi et al. 2002). A correspondence can be established between reference frame parameters that are origin, scale and orientation and the parameters $\mathbf{T}$, s and $\mathbf{R}$ and their rates. Equation A1 highlights the relationship between velocities and the time evolution of the reference frame parameters. A rotation matrix is required to transform velocities from a geocentric frame to a topocentric frame. Restricted to the up component, this matrix is given by:

$$\mathbf{G}(\lambda_{ij}, \phi_{ij}) = [\cos(\phi_{ij})\cos(\lambda_{ij}) \quad \cos(\phi_{ij})\sin(\lambda_{ij}) \quad \sin(\phi_{ij})] \tag{A2}$$

where $(\lambda_{ij}, \phi_{ij})$ are the longitude and latitude of station i, j.

The reference frame orientation is conventionally defined (McCarthy and Petit 2004). The time evolution of the orientations of the ITRFs were defined by aligning the velocity field orientation, either with respect to a geophysical velocity field (Altamimi et al. 2002) or with respect to a previous realization of the reference system (Altamimi et al. 2007). Current realizations of the frame orientation are believed to be at the level of 2.0 mm/year (Altamimi et al. 2003, 2007). However, the issue of orientation solely concerns horizontal velocities and will not be investigated in this study. In contrast, origin and scale are provided by space geodesy measurements. The two latest realizations of the ITRS based their definition of origin on SLR and their scale on either SLR and Very Long Baseline Interferometry (VLBI) data for ITRF2000, or VLBI data only for ITRF2005. Both were found to differ by 0.5 ± 0.3 mm/year (0.08 ± 0.05 ppb/year) for the scale and 1.8 ± 0.3 mm/year for the drift of origin in the Z component (Altamimi et al. 2007).

Appendix B: Error estimation for the reference frame origin

Using Eqs. 1 and 3, the empirical variance of the regional sea level trend with respect to the global averaged sea level trend is given by:

$$f(\Delta\dot{T}, \Delta\dot{d}) = \frac{1}{N-1}\sum_{i=1}^{N}\left(\sum_{j=1}^{n_i} p_{i,j}(v_{i,j}^{tg} + v_{i,j}^{up} + \Delta\dot{d} + \mathbf{G}(\lambda_{ij}, \phi_{ij})\cdot\Delta\dot{\mathbf{T}}) - \frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{n_k} p_{k,l}(v_{k,l}^{tg} + v_{k,l}^{up} + \Delta\dot{d} + \mathbf{G}(\lambda_{kl}, \phi_{kl})\cdot\Delta\dot{\mathbf{T}})\right)^2 \tag{A3}$$

The scale term cancels out so that function f is not dependent of the scale rate error. Indeed, any scale rate error affects the geocentric sea level rise at each station and the mean sea level rise by the same constant term. Equation A3 becomes:

$$f(\Delta\dot{T}) = \frac{1}{N-1}\sum_{i=1}^{N}\left(\left[\sum_{j=1}^{n_i} p_{i,j}(v_{i,j}^{tg} + v_{i,j}^{up}) - \frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{n_k} p_{k,l}(v_{k,l}^{tg} + v_{k,l}^{up})\right] + \left[\sum_{j=1}^{n_i} p_{i,j}\mathbf{G}(\lambda_{ij}, \phi_{ij}) - \frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{n_k} p_{k,l}\mathbf{G}(\lambda_{kl}, \phi_{kl})\right]\cdot\Delta\dot{\mathbf{T}}\right)^2 \tag{A4}$$

Nullifying the first derivatives of Eq. A4 with respect to translation parameters and using Eq. A2 yields:

$$\sum_{i=1}^{N}\begin{pmatrix} (\gamma_x^i)^2 & \gamma_x^i\gamma_y^i & \gamma_x^i\gamma_z^i \\ \gamma_x^i\gamma_y^i & (\gamma_y^i)^2 & \gamma_y^i\gamma_z^i \\ \gamma_x^i\gamma_z^i & \gamma_y^i\gamma_z^i & (\gamma_z^i)^2 \end{pmatrix}\begin{pmatrix}\Delta\dot{T}_x \\ \Delta\dot{T}_y \\ \Delta\dot{T}_z\end{pmatrix} = -\sum_{i=1}^{N}\alpha_i\begin{pmatrix}\gamma_x^i \\ \gamma_y^i \\ \gamma_z^i\end{pmatrix} \tag{A5}$$

With

$$\alpha_i = \sum_{j=1}^{n_i} p_{i,j}\cdot(v_{i,j}^{tg} + v_{i,j}^{up}) - \frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{n_k} p_{k,l}\cdot(v_{k,l}^{tg} + v_{k,l}^{up}) \tag{A6}$$

And

$$\begin{pmatrix}\gamma_x^i \\ \gamma_y^i \\ \gamma_z^i\end{pmatrix} = \sum_{j=1}^{n_i} p_{i,j}\begin{pmatrix}\cos(\phi_{ij})\cos(\lambda_{ij}) \\ \cos(\phi_{ij})\sin(\lambda_{ij}) \\ \sin(\phi_{ij})\end{pmatrix} - \frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{n_k} p_{k,l}\cdot\begin{pmatrix}\cos(\phi_{kl})\cos(\lambda_{kl}) \\ \cos(\phi_{kl})\sin(\lambda_{kl}) \\ \sin(\phi_{kl})\end{pmatrix} \tag{A7}$$

The translation rate error that solves the system of Eq. A5 minimizes the function f. α_i are the only coefficients that are dependent on the height velocities and tide gauge trends. Also, origin rate error is linearly dependent on these coefficients and so is linearly dependent on geocentric sea level trends at an individual station. As a consequence, it is possible to compute the variance of the translation vector using covariance propagation.

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