GEOID UNDULATION DIFFERENCES BETWEEN GEOPOTENTIAL MODELS

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Abstract. Three geopotential models (OSU91A, GEM-T3, and GRIM4-C2), available in 1991, have been compared in several ways. The models have been differenced to find the geoid undulation difference are on the order of 1 m in land areas and 30 cm in ocean areas with extreme differences reaching 6 m. The models were also evaluated, augmented by higher degree terms, when necessary, through comparisons with undulations at Doppler and GPS positioned stations. The undulation difference at the Doppler stations was ±1.57 m with no significant difference between models. Using 4 GPS test areas, differences were seen between the various models. A final comparison was made between geoid undulations implied by a Geosat 17 day cycle and undulations from the three models. The OSU91A model performed best having a difference standard deviation of ±34 cm.

1. Introduction

The geoid undulation is considered to be the separation between the equipotential surface that represents a mean ocean surface and a reference ellipsoid. The geoid is considered by many to be defined in both the land and ocean areas. The original technique to determine geoid undulations was the Stokes' integral (Heiskanen and Moritz, 1967). This procedure required a global set of gravity anomalies. Since such a global set does not exist the implementation of the Stokes' integral only has been done with a great deal of approximation. An alternate method to calculate geoid undulations is through potential coefficients (C_{nm}, S_{nm}) that are used to describe the Earth's gravitational potential. In the late 1950's and continuing since then, the determination of the potential coefficients to higher and higher degrees has been a common goal of a variety of geodetic groups. These determination have been made from the analysis of the perturbations of the orbits of artificial satellites, and with a combination of such information with surface gravity data, and relatively recently with satellite altimeter data. These solutions have been carried out in a variety of ways leading to models that are complete up to degree 50 in some cases, and up to degree 360 in others.

It is of interest to compare the various geopotential models to see the differences in them. In doing such comparisons it is important that nearly current, in time, models be compared. Comparison with models that are more than several years apart in development may reflect the use of different data sets rather than inherit comparisons of different methods.

The purpose of this paper is to note a number of current geopotential models and to show comparisons between these models.

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2. Potential Coefficient Models

This article will examine the three geopotential models shown in Table I.

The TEG-2 model (Tapley, 1991, private communication) was also examined by (Rapp, Wang, and Pavlis, 1991a,b). The model is currently being improved so that it was not used in the comparisons reported here.

TABLE I
Potential coefficient models considered

Designation	Maximum degree	Reference
OSU91A	360	Rapp et al. (1991)
GEM-T3	50	Putney *et al.* (1991)
GRIM4-C2	50	Reigber *et al.* (1991)

3. Results of Comparisons

Given several potential coefficient models there are a number of ways in which they can be compared. First is their mutual comparison while second is their evaluation against some external standard. The mutual comparison may be done in a spatial (geographic) sense or in a spectral (by harmonic degree). One may also examine the correlation between different models (through correlation coefficients) or by computing the percentage difference between two coefficient sets by degree, and over the whole coefficient space. Some of the comparisons with external information require the fields that are complete to degree 50 to be augmented with the coefficients from degree 51 to 360. This will be done for the tests reported here with the OSU91A coefficients. The length of this paper does not permit a complete set of analysis to be presented. However a representative set of comparisons and evaluations will be given.

Table II shows the root mean square differences (RMS) between selected geopotential models computed by differencing the potential coefficient sets and evaluating the following equation

$$\Delta N = a\left[\sum_{n}\sum_{m}(\Delta C_{nm}^2 + \Delta S_{nm}^2)\right]^{1/2} \tag{1}$$

TABLE II
RMS geoid undulation difference implied by selected geopotential models to degree 50. Units are cm

Model	OSU91A	GEM-T3
OSU91A	–	47
GEM-T3	47	–
GRIM4-C2	70	65

In Equation (1) ΔN is the RMS undulation difference, a is the equatorial radius, ΔC and ΔS are the differences of the fully normalized potential coefficients.

Examination of the global undulation differences shows that the models agree better in the ocean areas than in land areas. To demonstrate this the RMS undulation difference has been computed in land and ocean areas using discrete undulation values, on a $1° \times 1°$ grid, from the models. The ocean area is defined where the mean elevation is negative between $\pm 70°$ latitude. The land cells are all remaining blocks. Specifically given in Table III and IV are the standard deviations of the undulation differences in the land and ocean areas.

TABLE III
Standard deviation of the geoid undulation differences in land areas. Units are cm

Model	OSU91A	GEM-T3
OSU91A	–	75
GEM-T3	75	–
GRIM4-C2	115	105

TABLE IV
Standard deviation of the geoid undulation differences in ocean areas. Units are cm

Model	OSU91A	GEM-T3
OSU91A	–	29
GEM-T3	29	–
GRIM4-C2	40	41

From Tables III and IV one sees that the undulation differences are considerably smaller in the ocean areas than in the land areas. This is primarily due to the inclusion of satellite altimeter data or products derived from altimeter data in the combination models. This information enters the combination model in such a way as to give consistent results from the various groups.

Although the models agree well on an overall basis there are significant discrepancies between them. Table V shows the value of maximum (in an absolute sense) differences between the models.

Rapp *et al.* (1991b), Figure 30, shows a plot of the geoid undulation difference between OSU91A and GEM-T3. The largest difference occurs in the Himalayan

TABLE V
Maximum absolute value of geoid undulation differences. Units are cm

Model	OSU91A	GEM-T3
OSU91A	–	446
GEM-T3	466	–
GRIM4-C2	646	538

area. Other large differences on land are seen in South America. Substantial (~2 m) differences in ocean areas are seen in the Mediterranean Sea and below $-60°$ latitude. These differences occur because no altimeter data in these areas was used in the GEM-T3 model.

The largest differences with the GRIM4-C2 model occur in the Antarctic region at two locations: $\phi = -75$, $\lambda = 328°$ and $\phi = -84°$, $\lambda = 342°$. These differences reach 5.3 meters when comparisons are made to GEM-T3 and 6.3 m when OSU91A is compared. Other large (~3 m) differences occur in the Himalayan area and in the Andes Mountains in South America.

The last comparison between the geopotential models is by undulation difference by degree. Figure 1 shows the differences up to degree 50 for three cases: (a) GRIM4-C2/OSU91A; (b) GRIM4-C2/GEM-T3; (c) GEM-T3/OSU91A. Comparison *c* shows smaller differences than the other two differences most probably due to the fact that the OSU solution started from a NASA (GEM-T2) potential coefficient model. One may also note the significant difference at degree 43 which is the resonance order for Geosat.

4. Model Evaluation

The geoid undulations implied by each potential coefficient model can be evaluated by comparing such undulations to external undulation estimates. In doing these comparisons the potential coefficients above degree 50 for any model were taken to be those of the OSU91A model to degree 360. Details of the methods of comparisons to be described here may be found in Rapp and Pavlis (1990) and Rapp *et al.* (1991b).

The first comparisons are with geoid undulations derived from stations whose positions were originally determined in the NSWC 9Z-2 satellite reference system using Doppler positioning techniques. Before comparisons were done, the Doppler positions were translated to a center of mass system and properly scaled. The mean difference, the standard deviation of the difference, and the number of stations used with a 4 m residual rejection criteria are given in Table VI.

TABLE VI
Comparison of geoid undulations implied by Doppler positioning with undulations from the augmented geopotential models

Model	Mean difference	Standard deviation	Number of stations
OSU91A	15 cm	1.58 m	1802
GEM-T3	12	1.57	1788
GRIM4-C2	7	1.57	1796

From Table VI one sees little difference in the models although the use of

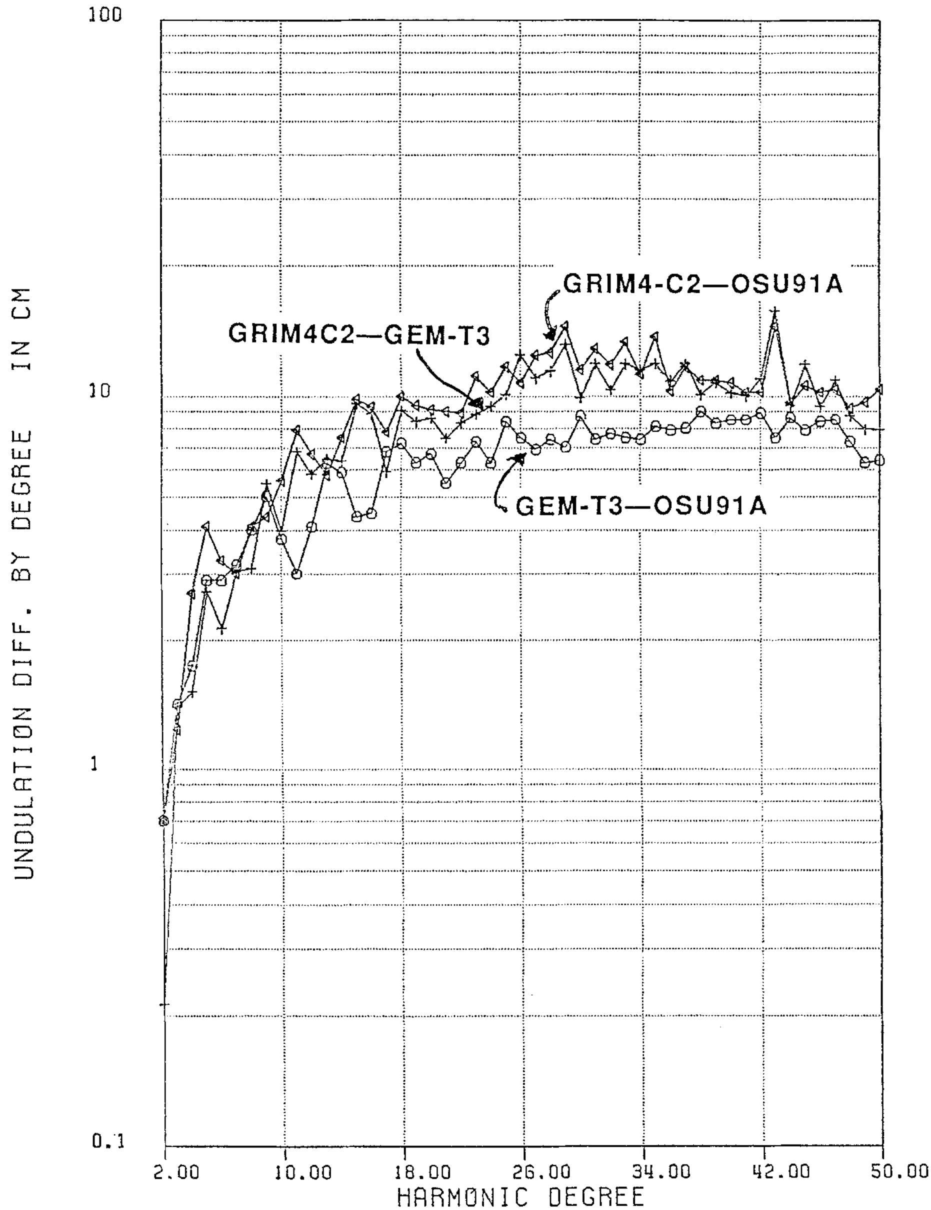

Fig. 1. Geoid undulation differences by degree for 3 geopotential models.

OSU91A enables 6 more stations to be accepted in the solution over that of the GRIM4-C2 model.

The next set of comparisons is at a set of stations, in four areas, whose positions were determined from GPS observations. The standard deviation of the differences along the traverses tested (described in Rapp and Pavlis, 1990) are given in Table VII.

TABLE VII
Standard deviation of the geoid undulation differences at GPS stations in four areas. Units are cm

Model	Area Europe	Canada	Australia	Tennessee
OSU91A	33	36	35	21
GEM-T3	42	36	35	23
GRIM4-C2	41	44	26	19

The results seen from Table VII present a mixed picture. For the European traverse the OSU91A model gives the best results while for the Australia traverse GRIM4-C2 is the best although it is poorest for the Canadian traverse.

Comparisons were also made at the GPS stations using undulation differences. This differencing removes some of the long wavelength error in the geopotential model. The results for these comparisons are shown in Table VIII. The results are given in the root mean square difference and is parts per million (ppm) of the distance between the stations.

TABLE VIII
Relative geoid undulation comparisons based on GPS positioning

	Area							
	Europe		Canada		Australia		Tennessee	
Model	RMS	ppm	RMS	ppm	RMS	ppm	RMS	ppm
OSU91A	23	3.6	9	6.6	22	5.3	26	3.9
GEM-T3	24	3.7	9	6.6	22	5.3	26	3.9
GRIM4-C2	24	3.8	19	8.0	23	5.4	26	3.8

Table VIII indicates that most solution are comparable although the GRIM4-C2 model does not perform well in the Canadian test.

The last evaluation was carried out by comparing a geoid undulation from the augmented geopotential model with a corrected sea surface height implied by Geosat altimeter data. The corrections used were those from the GEM-T2 orbit improvement process and the reduction of a sea surface height to geoid undulation using the OSU91 sea surface topography model (Rapp *et al.*, 1991b). The comparisons were done for one exact repeat mission (ERM 7) on an ocean wide basis and in a more restricted basis excluding data below $-60°$ latitude and in the Mediterranean Sea. The latter comparison was carried out because data deletion in the GEM-T3 and GRIM4-C3 model development had an impact on the accuracy in these comparisons. Table IX gives the standard deviation of the difference between the two geoid undulation estimates and the number of residuals that exceed 1.5 m.

TABLE IX
Comparison of the model and Geosat (ERM 7) implied geoid undulations

	Ocean area		Restricted area	
Model	Std. dev.	No. ≥ 1.5 m	Std. Dev.	No. ≥ 1.5 m
OSU91A	34 cm	4505	34	3656
GEM-T3	49	16204	40	6054
GRIM4-C2	59	21917	49	9105

Table IX indicates that the OSU91A model gives the best fit to the altimeter data. This may be a direct effect of the use of the Geosat altimeter data in the determination of the coefficients to degree 50 in the 91A model. The GRIM4-C2 geoid undulations show the poorest agreement with the Geosat data. In the case of GEM-T3 and GRIM4-C2 better agreement is seen when the comparison is made in the restricted area. No such improvement is seen when the OSU91A model is used indicating the same level of agreement in the total ocean areas and the restricted area.

5. Conclusions

This paper has carried out a comparison and evaluation of three current (1991) geopotential models. The word current (1991) is important here because the development of geopotential models is rapid. We also emphasize current because a comparison of models developed at substantially different time periods reveals only information on the differences in the models and not on the accuracy of the models.

Of the models examined in this paper one is complete to degree 360 while two are complete to degree 50. The model *comparisons* were carried out using the models to degree 50. The model *evaluation* was carried out with the individual models augmented by the OSU91A coefficients from degree 51 to degree 360.

The global geoid undulation difference is on the order of ±60 cm. However the differences are smaller (±35 cm) in the ocean areas than in the land (±85 cm). There are some areas, primarily land, in which the undulation differences between the models reaches 6 m. The larger differences occur in the mountainous regions of Asia and South America and in Antarctica.

The undulation differences between the models were also studied by spherical harmonic degree. This spectral information shows, for example, the undulation difference at degree 26 is 7.5 cm for OSU91A/GEM-T3 and 12 cm for GRIM4-C2/GEM-T3.

The evaluation of the models in terms of geoid undulation quantities was carried out through comparisons with Doppler and GPS derived geoid undulations. The Doppler comparisons are not sufficiently accurate to see differences in the models.

However the GPS comparisons do reveal that some models fit better in a certain geographic region.

The last evaluation was carried out using Geosat altimeter data. These comparisons show that the OSU91 model fits the altimeter implied geoid undulations better than the other two models with a rms difference of ±34 cm. Improvements in the fits occur when areas are deleted in the comparison in which no altimeter data was used in the computation of the model.

This paper has emphasized the comparison of geopotential models through geoid undulations. One could choose other quantities, (for example, gravity anomalies) for such comparisons. The evalution of the models has been done with geoid undulation information. Other important procedures are available for the evaluation of the models most important being satellite orbit fits with the various geopotential models.

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References

Putney, B., *et al.*, 1991, Earth Gravity Model Development at NASA/GSFC: Preliminary Results from GEM-T3 and GEM-T3S, EOS, (abstract), 72(17), p. 89.

Rapp, R. H., and Pavlis, N. R.: 1990, 'The Development and Analysis of Geopotential Coefficient Models to Spherical Harmonic Degree 360', *Journal of Geophysical Research* **95**(B13), 21,885–21,911.

Rapp, R. H., Wang, Y. M., Pavlis, N. K.: 1991a, 'Geoid Undulation Differences Between Geopotential Models', presentation at the meeting of the European Geophysical Society, Wiesbaden, Germany, April.

Rapp, R. H., Wang, Y. M., Pavlis, N. K.: 1991b, 'The Ohio State 1991 Geopotential and Sea Surface Topography Harmonic Coefficient Models', Report 410, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, 1991.

Reigber, C. *et al.*: 1991, 'GRIM4-C1, C2: Combination Solutions of the Global Earth Gravity Field', presentation at the meeting of the European Geophysical Society, Wiesbaden, Germany, April. (published in this issue, pp. 381–393).