#### **Chapter 46**

# The Map of Lazarus (1528) of Hungary and the Ptolemian Projection

Gábor Timár<sup>1</sup>, Gábor Molnár<sup>2,1</sup>, Balázs Székely<sup>2,1</sup> and Katalin Plihál<sup>3</sup>

- <sup>1</sup> Dept. of Geophysics and Space Science, Eötvös University, Budapest, Hungary timar@ludens.elte.hu
- <sup>2</sup> Institute of Photogrammetry and Remote Sensing, Vienna University of Technology, Austria
- <sup>3</sup> Map Archive, National Széchenyi Library, Budapest, Hungary

# Abstract

The strange orientation of the map of Lazarus (1528) has been a subject of a long debate of Hungarian cartographers in the 20th century. In this map, northeast is up, instead of the normal and traditional orientation where the north is up. It was long ago supposed that this orientation is a result of the local/regional usage of the Ptolemian projection of the world maps of the age of the map construction. If a Ptolemian conic projection is defined in the GIS environment with the parameters of  $\Phi_1$ =0°,  $\Phi_2$ =64° and  $\Lambda_0$ =90° (from Greenwich), interestingly enough, the map can be rectified and the resulted image has right angles at its corners and all sides are horizontal or vertical in the Ptolemian coordinate system but not, of course, in the modern ones. The linear rectification errors in this projection are more or less equal to the quadratic ones in fitting to modern coordinate systems eg. to a UTM zone. It implies that the above projection can be considered at least as a substituting one or even the real projection of the Lazarus map. If we consider this projection as a Ptolemian one, it can be deduced that Lazarus used the equidistant conic projection with two standard parallels: the Equator and the Northern Circle, which is more or less the same as the mysterious Parallel of Thule in the maps of Ptolemy. In the map, however, the main directions are rotated by 90°; the grid north points to the original



ICA Symposium on Cartography for Central and Eastern Europe Vienna, 16-17 February 2009 III: Georg Gartner & Felix Ortag (Eds.): Proceedings of the First ICA Symposium for Central and Eastern Europe 2009, Vienna University of Technology, 2009. left indicated by the word 'Occidens' (west), which is considered as an error of the press preparation.

**Keywords:** Lazarus Secretarius, Ptolemian projection, historical maps, Pannonian Basin, georeference, GIS

#### 46.1 Introduction

The map of Lazarus (1528) is the first map showing the historical Hungary and its surroundings in one cartographic product consisting of four sheets. The orientation of this map, however, is strange for the modern reader; the geographic north is rotated to right by cca. 40-45 degrees (Fig. 46.1). There were a lot of explanations of this rotation in the past decades. The debate followed two line of thoughts. According the first one, the reason would be the usage of the world projection of Ptolemy (cf. Snyder, 1987; 2007). The other opinion is that either the map or its orientation is not related to anything like projections, geodesy and modern topography (eg. Bede, 1987; Lotz, 1988; Fleck, 2003). These authors consider the selection of the orientation as a tool that provides the most comfortable display of the territory of the country.

The first explanation appeared earlier: it was first presented by Cholnoky (1943), who pointed out that the orientation of the Lazarus map is natural if we put this piece of land to the conic world projection of Ptolemy (Fig. 46.2). In the terms of the modern cartography, the network north in this projection is up, while the geographic north considerably differs from it. This idea was supported later by Fodor (1952) and Irmédi-Molnár (1958; 1964), too.

The following literature focused mainly on the reconstruction of the geographic coordinate network at parts of the map (Hrenkó, 1974; Érdi-Krausz, 1976; 1982). Fleck (1979) provides the first analytical review of the location coordinate data available of the time of Lazarus. The distortion of the map shows considerable regional differences, which led to opinions denying any projections of the map of Lazarus. The systematic projection analysis, based on distortions represented by the Tissot indicatrices of the map was made by Stegena (1976; 1982).

There is one point that is accepted by all researchers: the basis of the survey was the itineraries (Cholnoky, 1943; Plihál, 1990, 2003; Török, 1996), whose onedimensional character excludes the geodetic approaches. In our previous work (Molnár et al., 2008), however, we showed that the map of Lazarus could be rectified with surprisingly good accuracy using the modern GIS tools. The GIS methods offer also a possibility to check, whether the above mentioned opinion of Cholnoky (1943) is correct and with what accuracy. For this, we have to have an assumption



**Fig. 46.1.** The map of Lazarus (1528), the first cartographic work of the whole historical Hungary and its surroundings.

about the way of creation of the Lazarus map. Besides, the results provide interesting additions to the practical modelling of the Ptolemian projection.



**Fig. 46.2.** The structure of the world maps of Ptolemian type and the location of the Lazarus map in it.

### 46.2 How would we make the map of Lazarus?

Besides of reading the literature, it is worth contemplating on how we could solve the surveying and drawing if *we* were in place of Lazarus with the available knowledge of the early 16th century. We have to know *some* control points to start the survey from. In a such big area that the Pannonian Basin is, these points have to be defined by geographic coordinates. Besides, we have to have a rule describing the location of the control points in the coordinate system of the map. These requirements are somewhat the historical equivalents to the 'first-order geodetic network' and the 'map projection'. However, the latter one was not defined by equations but by geometric drawing rules.

Could have been point coordinates known for Lazarus? Of course, the coordinate list of Ptolemy (83-161), with several thousands of locations (almost fifty of them lying within the extents of Lazarus' map) were available for him, as well as the lists of Regiomontanus of Alphonsus. From Ptolemy's *Geographia* and *Almagest*, it was also known, how to place these coordinate pairs to the cosmographic (world) map used since the ancient times. Several points, located in *Pannonia Superior* and *Inferior*, *Noricum*, *Illyricum*, *Dacia* and *Sarmatia* (Fehér, 2004), could be drawn in the empty paper or pergamen as a starting step of the map making. The details between these control point could have been based on the itineraries (Plihál, 1990; Török, 2007a), which was the ancient counterpart of the modern traversing, along the roads of the country. Some fortresses appear doubled in the map; the traversing

could be a possible explanation to that; their locations resulted to different places according to different traverse lines, all affected by local errors. The map maker hasn't decided, which location is the 'correct' one – he had no tools for cross-checking – so he drawn both in the map.

Based on the above considerations and speculations, we argue any statements about the 'projectionless' character of the map, based on the local errors of the detailed surveys. Of course, the coordinates of the control points are also burdened by errors (Fleck, 1979), and the errors of the survey method increase these, resulting the total error of the map positions. If the accuracy of the georeference is not worse than that (and, as we see later, it is not worse), we can suppose the application of a map projection. Let's see, could it be the Ptolemian projection, which was used at the world maps of this time?

## 46.3 The Ptolemian projection in the GIS systems

If we assume that the map of Lazarus has the Ptolemian projection and we want to support it by GIS tools we have to know the type of the projection and also its parameters. The type is obviously the equidistant conic projection (Snyder, 1987). In our opinion, the scale factor (the character of one or two standard parallels used) is not important. What is vital, is the peak angle of the cone, therefore the angles between the meridians. We used the ER Mapper GIS software for the analysis. It enables to define the equidistant conic projection with two standard parallels. We could try to estimate the location of the standard parallel using the meridian angles (Timár et al., 2003). The problem with this method is that the longitude values in the Ptolemian maps are considerably over-estimated (cf. Török, 2007b).

To estimate the parameters of the projection (the location of the projection center and the standard parallels), we can use the map of Lazarus itself. In our recent work (Molnár et al., 2008) we defined a database of more than 600 control points about the written locations in the map. These points are defined (1) in the image coordinate system of the scanned map image and (2) by the modern geographic coordinates of the settlements. The Ptolemian projection type was tried with several standard parallel pairs and with different projection centers. Using the tested parameters, the geographic coordinates of the database locations were recomputed to planar coordinates. Linear function parameters were estimated between these planar coordinates and the known image coordinates of the points. As a result, we found that the position of the standard parallels affects the corner angles of the resulted rectified map, while the projection center affects its rotation. Both are important for the projection parameters; we were searching a parameter set providing a rectified map with right



**Fig. 46.3.** Rectification of the map of Lazarus (a) with incorrect standard parallels and projection center (b) with incorrect standard parallels but correct central meridian (c) with correct standard parallels but incorrect central meridian (d) with correct parameters.

angles in its corners and its border lines are horizontals and verticals (Fig. 46.3). This result appears when the projection parameters are the following:

 $\Phi_1 = 0^{\circ}$ 

 $\Phi_2 = 64^{\circ}$ 

so the first standard parallel is the Equator and the second is the one that marked as 'Parallel of Thule' in the Ptolemian maps. This latter one is, in our opinion, the ancient version of the polar circle. Its latitude slightly differs from the modern value of 66.5 degrees.

The latitude definition of Ptolemy is basically not explained in degrees but the length of the longest daylight of the year. Therefore, no latitudes north of the polar circle, are given. It can be imagined that the maximum daylight length of 24 hours were detected at this latitude, because of

• the planetary precession (a periodic change of the angle between the rotation axis and the Ecliptic, which is now 23.5 degrees; between 20 and 24.5 degrees with a period of 40,000 years), which altered the position of the polar circle by about half degree since the 2000 years of the Ptolemian observations; • the real extents of the Sun in the sky, which is about half a degree, and

• the refraction, also causing ca. half a degree.

These three factor can explain only 1.5 degrees of the real 2.5 degrees, so our explanation that the 'Parallel of Thule' is the ancient polar circle is not evidenced. Although it not relevant in our analysis, it can be hypothesized that these two parallels were selected for standard ones because they are the locations of the shortest (at the Equator) and longest (at the northern polar circle and northwards) period of the maximum daylights.

If these two parallels are selected for standard ones, the corners of the rectified map are right angles, not depending on the location of the projection center. The projection center affects the angle between the map bottom and top lines to the horizontal. The longitude of projection centers, which provides horizontal top and bottom lines is:

 $\Lambda_0$ =90° (from Greenwich)

At first sight this figure is not surprising as it is expected that the central meridian is in the halfway of the known world of 180 degrees longitude range in the Ptolemy's maps. However, these maps have the prime (westernmost) meridian not at Greenwich, but more to the west, at the westernmost point of the Old World, in the Canary Islands, practically at a meridian that was later called Ferro. The central meridian from Ferro is no longer 90 but 107-108 degrees.

If we look a Ptolemian world maps (Fig. 46.2), we see the Old World from the Canary Islands to Japan, which is a longitude range of 160 degrees. Most of the maps of Ptolemy's type indicate this range as 180 degrees but some of them (see eg. Török, 2007b) take a larger angle. Consequently, all longitude values have a longitude-dependent distortion. The longitude of *Aquincum* (Roman settlement at Budapest) should be 36-37 degrees from Ferro, while its position is 43 degrees in the Ptolemian coordinate list. Therefore, the longitude distortion is larger in the western part of the *oikoumene* (oikouµévη), the Old World. This results that the optimal position of the projection center is not ninety degrees from Ferro.

Nevertheless in case of the equidistant conic projection, defined with the above parameters, the rectified version of the Lazarus map has right angles in the corners and horizontal and vertical borders (Fig. 46.4).

Further explanations are needed about the direction descriptions in the map. Namely, the western direction ('*Occidens*') is up, which is mostly irregular. A speculative explanation can be the following: The original manuscipt could be in Landscape format, with the north up. During the press preparation works it was re-designed to Portrait format while the title and the coat of arms were also placed in the map. These two elements divided the original structure into three parts. The printer, who placed the direction descriptions to the press form, was not aware of the above modifications or the content of the accompanying text. This latter informa-



**Fig. 46.4.** The rectified map of Lazarus with the modern vector data showing the latitude-longitude grid and the drainage network. Borders of the historical Hungary are indicated by red line.

tion says 'If you turn the map correctly to the directions, you'll see, which town lies to west or to east from each other, or similarly to south or to north". This message would be not needed at all, if there are direction descriptions in the map (Plihál, 1990).

# 46.4 Rectification of the Lazarus map in the Ptolemian projection

It is worth comparing the rectification accuracy of the Lazarus map in the above defined Ptolemian projection (henceforth referred to as LP-projection) and in the modern projections (Molnár et al., 2008). The *linear* rectification to the LP-projection can be made with similar accuracy as the *quadratic* rectification to the modern projections. This advocates the concept of the usage of the LP-projection at the map of Lazarus. The average accuracy of the rectification is 15-20 kilometers. The errors are considerably lower in the central part of the country. Around some cities where astronomical observatories could be present in the second part of the 15th century (Vienna, Buda, Várad [Oradea], Gyulafehérvár [Alba Iulia]) there is almost zero error. In the external parts of the map, at the Adriatic coast, at the lower reach of the Danube and Sava Rivers and in Eastern Transylvania, the accuracy is lower, errors of ca. 30 km occur. It has to be noted that these zones belonged to chapters of the Ptolemian point list other than the Pannonian one (*Dacia, Moesia, Illyria*).

Finally we mention that the latitude of the projection center is insignificant in case of correct setting of the standard parallels, therefore. the false eastings and false northings of the projection center were set to zero. As there is no indication of the projected coordinates in the map of Lazarus, the coordinates in the LP-projection are merely auxiliary variables; those values depend on the latitude of projection center, but it does not affect the rectification in the modern coordinate system.

#### 46.5 Summary

Based on a control point list, containing around six hundred localities in the map of Lazarus, we concluded that the map can be rectified in the equidistant conic projection (the first Ptolemian projection) with the following parameters:  $\Phi_1=0^\circ$ ;  $\Phi_2=64^\circ$ ;  $\Lambda_0=90^\circ$  (from Greenwich). Applying this projection, the rectified map has right angles in its corners and horizontal and vertical borders. The latitude of the projection center is indifferent, the projected coordinates of the center were set to zero. The average accuracy of the rectification is 15-20 km, the maximum errors are slightly over 30 km. If we accept that the 'Ptolemian projection' was used in that period, then the map of Lazarus provides indirect evidence that the two standard parallels of this conic projection were the Equator and the northern polar circle (or, the perhaps similar, 'Parallel of Thule').

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